Accelerated Motion

Acceleration is the change in velocity per unit of time.

$$\vec{a} = \frac{\Delta \vec{V}}{t}$$

units

or $\boxed{\vec{a} = \frac{\vec{V}_f - \vec{V}_i}{t}}$ $\left[\frac{m}{s^2}\right] = \frac{\left[\frac{m}{s}\right]}{[s]}$

Say $a = 9.8\ \frac{m}{s^2}$ $\Rightarrow$ every second the $\vec{V}$ changes by $9.8\ \frac{m}{s}$

Example:

A car traveling at $50\frac{km}{h}$ takes 3.5 s to come to a stop. What is the car's acceleration?

Let fwd be +

$t = 3.5\ s$

$\vec{V}_f = 0$

$\vec{V}_i = 50\frac{km}{h}$) ÷3.6

$= 13.9\frac{m}{s}$

$\vec{a} = ?$

$a = \frac{V_f - V_i}{t}$

$= \frac{(0)-(13.9)}{3.5}$

$= -3.97\ \frac{m}{s^2}$

∴ the car accelerates $3.97\frac{m}{s^2}$ [bwd]

A proton in an accelerator starts off going $8.9\times10^6\ \frac{m}{s}$ [W]. It then accelerates at $2.5\times10^9\frac{m}{s^2}$ [E] for 16 ms. What is its final velocity?

+ W ⟷ E −

$8.9\times10^6\frac{m}{s}$

V_f

$a = -2.5\times10^9\frac{m}{s^2}$

$V_i = 8.9\times10^6\frac{m}{s}$

$t = 16\ ms$

$= 0.016\ s$

$V_f = ?$

$a = \frac{V_f - V_i}{t}$

$at = V_f - V_i$

$V_f = V_i + at$

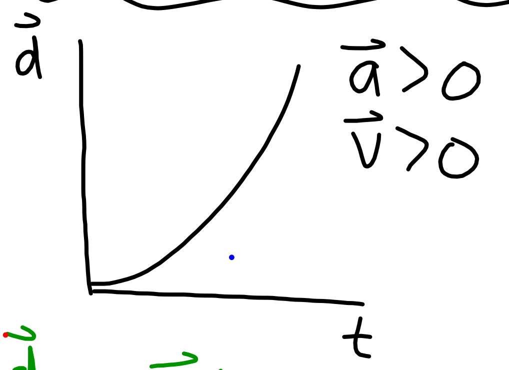
$= (8.9\times10^6) + (-2.5\times10^9)(0.016)$

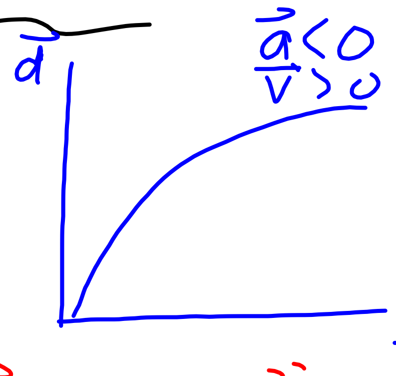
$= 8.9\times10^6 - 4\times10^7$

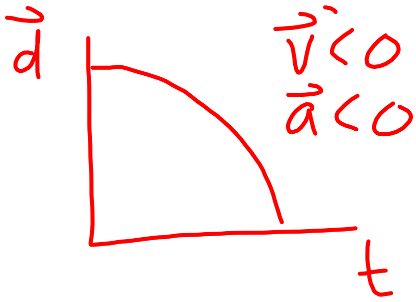
$= -3.11\times10^7\ \frac{m}{s}$

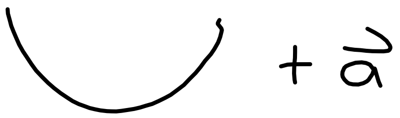
∴ the final $\vec{v}$ of the proton was $3.1\times10^7\ \frac{m}{s}$ [E]

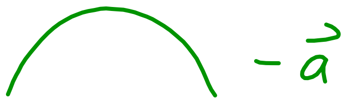
Graphing Acceleration











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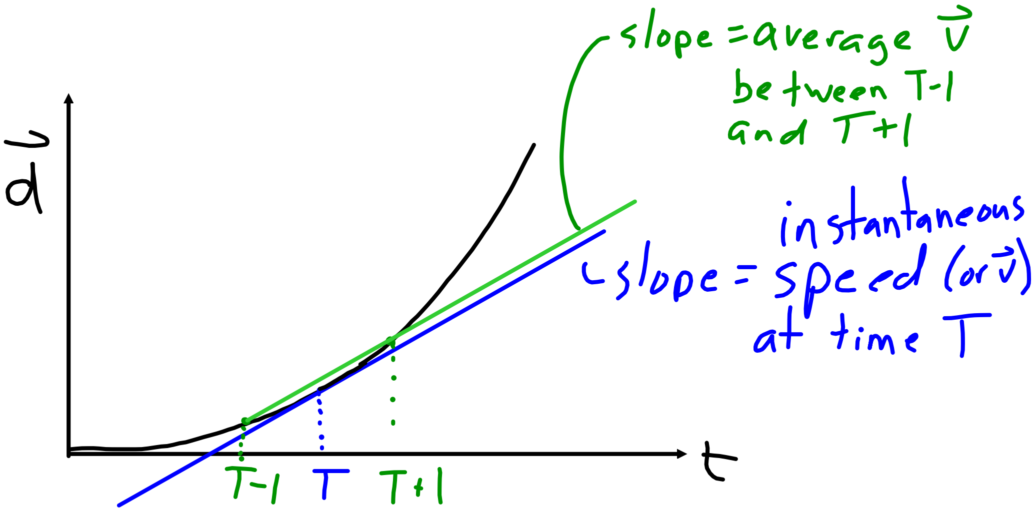
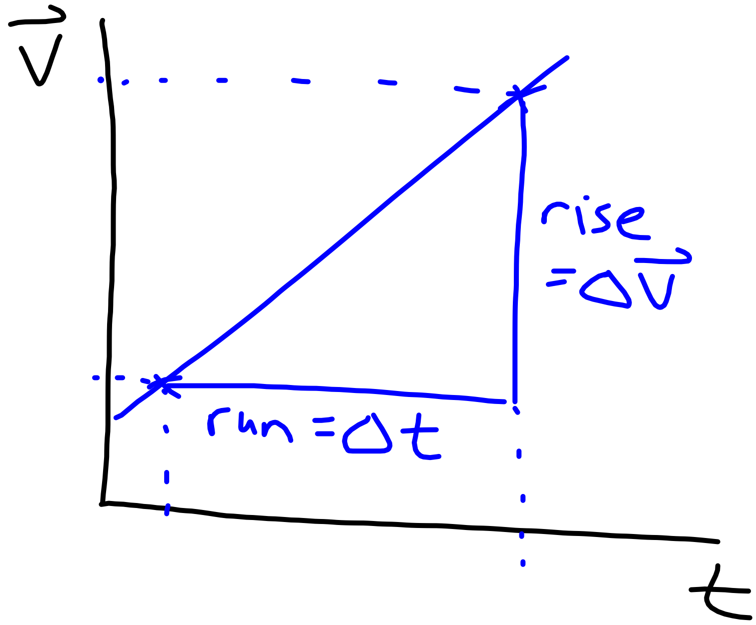


The average $\vec{v}$ between 2 points is the same as the instantaneous velocity at its midpoint.
(ONLY IF $\vec{a}$ IS CONSTANT)



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta \vec{v}}{\Delta t} = \vec{a}$$

t (s)	$\vec{d}$ (m)	$\vec{v}$ (m/s)
0	0	—
1	2	4
2	8	8
3	18	12
4	32	16
5	50	—

$$\vec{V}_1 = \vec{V}_{0 \to 2} = \frac{\Delta \vec{d}}{\Delta t} = \frac{8-0}{2-0} = 4\ \frac{m}{s}$$

$$\vec{V}_2 = \vec{V}_{1-3} = \frac{18-2}{3-1} = 8\ \frac{m}{s}$$

$$V_3 = V_{2 \to 4} = \frac{32-8}{4-2} = 12\ \frac{m}{s}$$

Kinematic Equations

$V_{AV} = \frac{d}{t}$ but... when $\vec{a}$ is constant $V_{AV} = \frac{V_i + V_f}{2}$

$\cancel{d}$ - 2

$\cancel{a}$ - 1

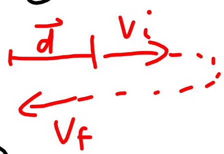
$\cancel{V_i}$ - 4

$\cancel{V_f}$ - 3

t

$$\frac{d}{t} = \frac{V_i + V_f}{2}$$

$$\boxed{d = \left(\frac{V_i + V_f}{2}\right) t} \quad (1)$$



$$\boxed{a = \frac{V_f - V_i}{t}} \quad (2)$$

Solve for V_f in (2)

$$V_f = V_i + at \qquad v_i = v_f - at$$

Sub into (1)

$$d = \left(\frac{V_i + (V_i + at)}{2}\right) t$$

$$= \frac{(2V_i + at)}{2} t$$

$$= \frac{\cancel{2} v_i t}{\cancel{2}} + \frac{at^2}{2}$$

$$\boxed{d = v_i t + \frac{1}{2} a t^2} \quad (3)$$

Similarly

$$\boxed{d = v_f t - \frac{1}{2} a t^2} \quad (4)$$

$[m]$ $\left[\frac{m}{\cancel{s}}\right][\cancel{s}]$ $\left[\frac{m}{\cancel{s^2}}\right][\cancel{s}]^2$

Solve for t in 2

$$t = \frac{V_f - V_i}{a}$$

Sub into 1

$$d = \left(\frac{V_i + V_f}{2}\right)\left(\frac{V_f - V_i}{a}\right)$$

$$= \frac{(V_i + V_f)(V_f - V_i)}{2a}$$

$$d = \frac{V_f^2 - V_i^2}{2a}$$

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$$2ad = V_f^2 - V_i^2 \Rightarrow \boxed{V_f^2 = V_i^2 + 2ad}$$

Example

A proton traveling at $2.5\times10^6\ \frac{m}{s}$ in an accelerator accelerates at $3.6\times10^8\ \frac{m}{s^2}$ in the other direction until it is going $8.9\times10^6\ \frac{m}{s}$ in the direction of its acceleration. What is its displacement during this time?

$$d=\left(\frac{v_i+v_f}{2}\right)t$$

$$a=\frac{v_f-v_i}{t}$$

$$d=v_it+\frac{1}{2}at^2$$

$$d=v_ft-\frac{1}{2}at^2$$

$$v_f^2=v_i^2+2ad$$

Let v_i be +

$v_i=2.5\times10^6\ \frac{m}{s}$

$a=-3.6\times10^8\ \frac{m}{s^2}$

$v_f=-8.9\times10^6\ \frac{m}{s}$

$d=?$

$$v_f^2=v_i^2+2ad$$

$$d=\frac{v_f^2-v_i^2}{2a}$$

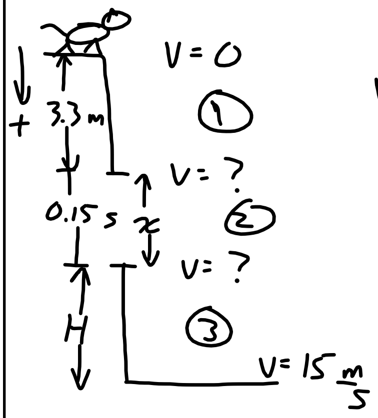
$$=\frac{(-8.9\times10^6)^2-(2.5\times10^6)^2}{2(-3.6\times10^8)}$$

$$=-1.01\times10^5\ m$$

A cat steps off the roof of a 3-storey building. A attentive physics student on the second floor determines that it took 0.15 s for the cat to fall the full height of the window, which is 3.3 m below the roof. The cat lands with a speed of 15 $\frac{m}{s}$.

a) How tall is the window?

b) How high is the bottom of the window (from the ground)?



①	②	③
$V_i = 0$ ✓	$V_i = 8.04 \frac{m}{s}$	$V_i =$
$d = 3.3\ m$ ✓	$\boxed{d = x}$	$\boxed{d = H}$
$V_f = 8.04 \frac{m}{s}$	$V_f =$	$V_f = 15 \frac{m}{s}$
$t =$	$t = 0.15\ s$	$t = ?$
$a = 9.8 \frac{m}{s^2}$ ✓	$a = 9.8 \frac{m}{s^2}$	$a = 9.8 \frac{m}{s^2}$

For section ①

$$V_f^2 = V_i^2 + 2ad$$
$$= 0^2 + 2(9.8)(3.3)$$
$$= 64.68$$
$$V_f = 8.04\ \frac{m}{s}$$

For Section ②

$$d = V_i t + \frac{1}{2} a t^2$$
$$x = (8.04)(0.15) + \frac{1}{2}(9.8)(0.15)^2$$
$$\doteq 1.206 + 0.110$$
$$x = 1.32\ m$$

Entire Fall...

$V_i = 0$

$V_f = 15 \frac{m}{s}$

$a = 9.8 \frac{m}{s^2}$

$$V_f^2 = V_i^2 + 2ad$$
$$15^2 = 0^2 + 2(9.8)d$$
$$\frac{225}{19.6} = d$$
$$d = 11.5\ m$$

$$d = 3.3 + 1.32 + H$$
$$H = 11.5 - 3.3 - 1.3$$
$$= 6.9\ m$$