

## Unit 5: Modern Physics

### Special Relativity

The relativity principle:

1. The laws of physics hold in any inertial frame of reference.
2. Light travels through empty space at a constant value,  $c$ , regardless of the initial frame of reference that the observer is in.

Consequences...

Consider a train moving at a speed  $v$ . A light source is placed at the bottom of the train and a detector immediately above it on the ceiling. One observer is in the train, while another is outside.

$$\begin{aligned}
 & \text{Diagram: A train moving to the right with velocity } v. \text{ Inside the train, a light source at position } l_0 \text{ emits a light pulse upwards towards a detector at position } l. \\
 & \text{An observer on the train sees the distance as } d = vt. \\
 & \text{An observer outside the train sees the light travel along a diagonal path from the source to the detector, forming the hypotenuse of a right triangle with legs } d \text{ and } l_0. \\
 & \text{The Pythagorean theorem gives } l^2 = l_0^2 + d^2. \\
 & \text{Since the speed of light is } c, \text{ the time for the light to travel is } l/c. \\
 & \text{Equating the time seen by the train observer (} l/v \text{) to the time seen by the outside observer (} l/c \text{), we get } l/v = l/c * \sqrt{1 - v^2/c^2}. \\
 & \text{Solving for } l, \text{ we find } l = l_0 / \sqrt{1 - v^2/c^2}.
 \end{aligned}$$

$$\boxed{t = \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

$$\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma \quad L = \frac{L_0}{\gamma}$$

$$m = m_0 \gamma$$

Mr. T. is on a train for 5 days.  
It is moving at 95% of c.  
You are waiting for him in class.  
How long will you need to wait?

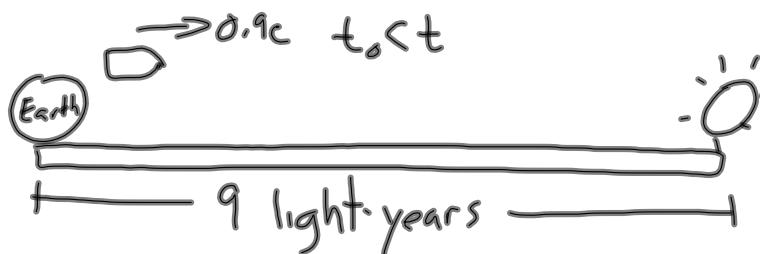
$$t_0 = 5 \text{ days} \quad t = \frac{5}{\sqrt{1 - \left(\frac{0.95c}{c}\right)^2}}$$

$$= \frac{5}{\sqrt{1 - 0.95^2}}$$

$$= \frac{5}{\sqrt{1 - 0.9025}}$$

$$= \frac{5}{\sqrt{0.0975}}$$

$$= \frac{5}{0.312} = 16.0 \text{ days}$$



$$t = 10 \text{ years}$$

$$L = \frac{L_0}{\gamma} \quad L_0 = 9 \text{ light years}$$

$$v = 0.9c \quad \gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$L = \frac{L_0}{\gamma}$$

$$= \frac{2.32}{\sqrt{1 - 0.9^2}} = \frac{2.32}{\sqrt{0.19}}$$

$$= \frac{1}{\sqrt{0.19}}$$

$$= \frac{1}{0.43}$$

$$= 2.32$$

$$\frac{1}{\sqrt{1 - \left(\frac{10}{300,000,000}\right)^2}}$$

$$E = mc^2$$

mass                          speed of light

energy stored as mass       $E = \gamma m_0 c^2$   
or  $\frac{m_0 c^2}{\sqrt{1 - (\frac{v}{c})^2}}$

The total energy of an object moving at a relativistic speed,  $v$ , is

$$E_T = \frac{m_0 c^2}{\sqrt{1 - (\frac{v}{c})^2}}$$

$$E_T = E_K + E_{rest}$$

$$\frac{m_0 c^2}{\sqrt{1 - (\frac{v}{c})^2}} = E_K + m_0 c^2 \rightarrow E_K = \frac{m_0 c^2}{\sqrt{1 - (\frac{v}{c})^2}} - m_0 c^2$$

How much  $E_K$  does a proton have when moving at  $1 \times 10^8 \frac{m}{s}$ ?

$$v = 1 \times 10^8 \frac{m}{s}$$

$$\frac{m_p}{m_0} = 1.67 \times 10^{-27} \text{ kg}$$

$$E_T = E_K + E_{rest}$$

$$E_K = E_T - E_{rest}$$

$$= \frac{m_0 c^2}{\sqrt{1 - (\frac{v}{c})^2}} - m_0 c^2$$

$$= m_0 c^2 \left( \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} - 1 \right)$$

$$= (1.67 \times 10^{-27}) (3 \times 10^8)^2 \left( \frac{1}{\sqrt{1 - (\frac{1}{3})^2}} - 1 \right)$$

$$= 9.11 \times 10^{-12} \text{ J}$$

~~$$E_K = \frac{1}{2} m v^2$$~~
~~$$= 9.35 \times 10^{-12} \text{ J}$$~~

$$E_K = \frac{1}{2} (m_0) v^2$$
~~$$= 9.39 \times 10^{-12} \text{ J}$$~~

$$1.88 \times 10^{-11} \text{ J} = 1.18 \times 10^{-9} \text{ eV}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$= 1.18 \times 10^3 \text{ keV}$$

$$= 1.18 \text{ MeV}$$

$$= 0.118 \text{ GeV}$$

Recall...

$$V = \frac{E}{q} \quad \therefore E = Vq$$

Let  $q = e$

$$\text{then } E = \underline{eV}$$

electron volts

1 eV is the energy gained by a charge of 1 e over a potential difference of 1 V

$$\therefore 1 \text{ eV} = (1.6 \times 10^{-19} \text{ C}) 1 \text{ V}$$

$$= 1.6 \times 10^{-19} \text{ J}$$

eV

keV

MeV

GeV