Unit 5: Modern Physics

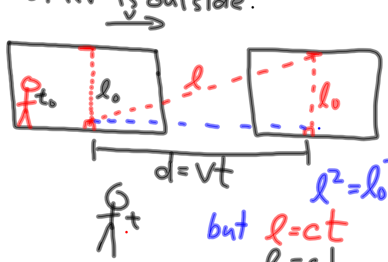
Special Relativity

The relativity principle:

1. The laws of physics hold in any inertial frame of reference.
2. Light travels through empty space at a constant value, c, regarless of the initial frame of reference that the observer is in.

Consequences...

Consider a train moving at a speed v. A light source is placed at the bottom of the train and a detector immediately above it on the ceiling. One observer is in the train, while another is outside.



$l^2 = l_o^2 + d^2$

but $l = ct$

$l_o = ct_o$

$d = vt$

$$(ct)^2 = (ct_o)^2 + (vt)^2$$

$$c^2t^2 = c^2t_o^2 + v^2t^2$$

$$c^2t^2 - v^2t^2 = c^2t_o^2$$

$$t^2\left(\frac{c^2 - v^2}{c^2}\right) = \frac{c^2t_o^2}{c^2}$$

$$t^2\left(1 - \left(\frac{v}{c}\right)^2\right) = t_o^2$$

$$t^2 = \frac{t_o^2}{1 - \left(\frac{v}{c}\right)^2}$$

$$\boxed{t = \frac{t_o}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}}$$

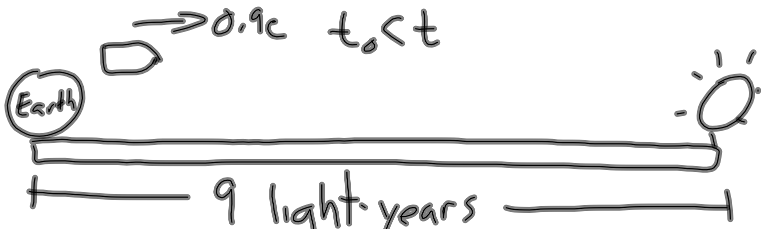
$$\boxed{t = \frac{t_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}} \qquad \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma \qquad L = \frac{L_0}{\gamma} \qquad m = m_0\gamma$$

Mr. T. is on a train for 5 days.
It is moving at 95% of c.
You are waiting for him in class.
How long will you need to wait?

$t_0 = 5$ days

$v = 0.95c$

$$t = \frac{5}{\sqrt{1-\left(\frac{0.95c}{c}\right)^2}} = \frac{5}{\sqrt{1-0.95^2}} = \frac{5}{\sqrt{1-0.9025}} = \frac{5}{\sqrt{0.0975}} = \frac{5}{0.312} = 16.0 \text{ days}$$



$t = 10$ years

$$L = \frac{L_0}{\gamma}$$

$L_0 = 9$ light years

$v = 0.9c$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-0.9^2}} = \frac{1}{\sqrt{0.19}} = \frac{1}{0.43} = 2.32$$

$$L = \frac{L_0}{2.32}$$

$= 3.88$ light-years

$$\frac{1}{\sqrt{1-\left(\frac{10}{300\,000\,000}\right)^2}}$$

$$E = mc^2$$

E: energy stored as mass

m: mass

c: speed of light

$$E = \gamma m_0 c^2$$

or $\dfrac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

The total energy of an object moving at a relativistic speed, v, is

$$E_T = \frac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

$$E_T = E_K + E_{rest}$$

$$\frac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = E_K + m_0 c^2 \rightarrow E_K = \frac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - m_0 c^2$$

How much E_K does a proton have when moving at $1\times10^8 \frac{m}{s}$?

$v = 1\times10^8 \frac{m}{s}$

$m_p = 1.67\times10^{-27}$ kg

m_0

$$E_T = E_K + E_{rest}$$

$$E_K = E_T - E_{rest}$$

$$= \frac{m_0 c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - m_0 c^2$$

$$= m_0 c^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right)$$

$$= (1.67\times10^{-27})(3\times10^8)^2\left(\frac{1}{\sqrt{1-\left(\frac{1}{3}\right)^2}} - 1\right)$$

$$= 9.11\times10^{-12}\ \text{J}$$

$\cancel{E_K = \frac{1}{2}mv^2}$

$\cancel{= 8.35\times10^{-12}}$

$\cancel{E_K = \frac{1}{2}(\gamma m_0)v^2}$

$\cancel{= 9.39\times10^{-12}\ \text{J}}$

$1.88\times10^{-11}\ \text{J} = 1.18\times10^8\ \text{eV}$

$1\ \text{eV} = 1.6\times10^{-19}$

$= 1.88\times10^{-11}$

$= 1.18\times10^5\ \text{keV}$

$= 118\ \text{meV}$

$= 0.118\ \text{GeV}$

Recall...

$V = \frac{E}{q}$ $\therefore E = Vq$

Let $q = e$

then $E = eV$

electron volts

1 eV is the energy gained by a charge of 1 e over a potential difference of 1 V

$\therefore 1\ eV = (1.6 \times 10^{-19}\ C)\ 1\ V$

$= 1.6 \times 10^{-19}\ J$

eV

keV

MeV

GeV