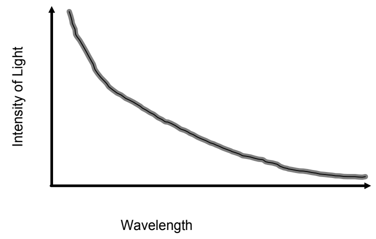
Introduction to Quantum Mechanics

Part 1: Blackbody Radiation
(the first sign of trouble)

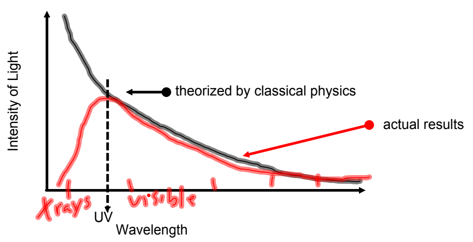
Up until the late 1800s, classical physics seemed to work well in all aspects of physics. Maxwell even managed to use classical physics in developing 4 important equations to describe how light behaved as a form of electromagnetic radiation. Then came the blackbody problem.

When an incandescent object is heated, it will start to glow red. As its temperature increases, the colour changes from red all the way to violet and even beyond. (As it turns out, all objects emit light, but not all of it is visible.) Any object that absorbs 100% of the light incident on it would appear black, and is thus called a blackbody. These objects also very efficient emitters of (electromagnetic) radiation.

When Maxwell's equations are used to explain why this radiation occurs, the theory is that vibration of the charges in the particles are responsible for the light given off. As temperature increases, particles vibrate faster, therefore the frequency of the light that is emitted should also be higher. If the intensity of the light given off was plotted vs the wavelength of the light, we would see this:



However, what we actually see is this:



Notice that the theory worked well until we got to UV radiation. This came to be known as the "ultraviolet catastrophe" since no classical theories could account for this. ("Catastrophe" might be a bit dramatic, but consider that this would have completely changed what physicists thought they knew.)

Part 2: Planck's Hypothesis

Max Planck proposed that the particles could only vibrate at specific quantities of energy. As a result, energy is given off in "chunks" called quanta and that energy contained by each quanta was proportional to the wavelength of the light.

$E = hf$ where h is Planck's constant (6.63×10^{-34} Js)
4.14×10^{-15} eVs

Classical physics assumed that energy can be found in a continuous amounts, where any value of E is possible. This is analogous to pushing a box up a ramp, where any height is possible.

Planck's quantum theory suggested that energy can only be found in multiples of the energy in each quanta ($E_n = nhf$). This is like lifting a box on a staircase; only specific heights are available (multiples of the height of one stair).

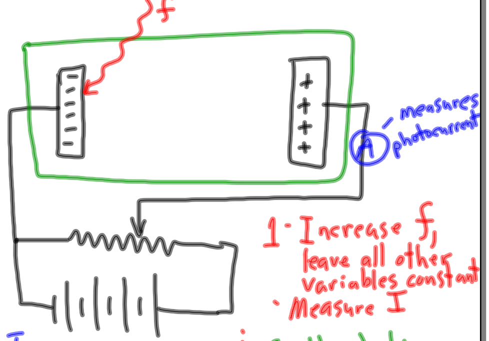
Note that since h is so small, E will be very small also. These quanta are very small bits of energy, so we don't even notice the effect on a large scale. It is only when we deal with small amounts of energy, like quanta of light energy (called photons) or individual atoms, that we really notice the effect of quantum theory.

Part 3: The Photoelectric Effect

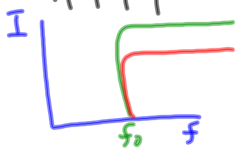
→ What it is...

When light shines on a metal surface, its energy can be absorbed by electrons orbitting the nuclei. If enough energy is absorbed, the electron can be ejected from the metal altogether.

Photoelectric effect experments:

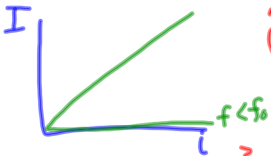


1- Increase f, leave all other variables constant
· Measure I

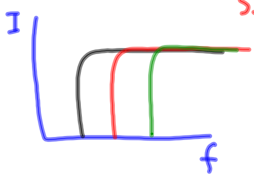


f_0 = threshold frequency

2. Increase intensity, i
measure I
$(f > f_0)$



3. Change the metal
change f
Measure I

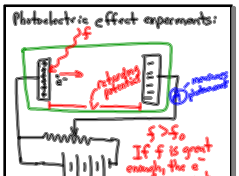


$E = hf$

E needed to eject e^-

$E = hf_0 = W$ ← work function

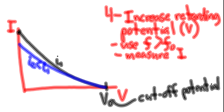
Photoelectric effect experments:



$f > f_0$
If f is great enough, the e^- may have enough E_k to still reach the − terminal

$E = Vq$

4- Increase retarding potential (V)
- use $f > f_0$
- measure I



V_0 cut-off potential

$$E_p = E_k + W$$

E_p: light energy (input); E_k: of e^-; W: to release e^- (output)

$$hf = E_k + W$$

Example:

a) What is the threshold frequency of aluminum?

b) What is the cut-off potential when 150 nm light is incident on the aluminum.

c) How much E_k do the photoelectrons have in this case?

a) $W = 4.20$ eV
$h = 4.14\times10^{-15}$ eVs

$E_p = \cancel{E_k} + W$
$hf_0 = W$
$f_0 = \frac{W}{h}$ $\left(\frac{eV}{eVs}\right)$
$= \frac{4.2}{4.14\times10^{-15}}$
$= 1.01\times10^{15}$ Hz

$\lambda = \frac{c}{f} = \frac{3\times10^8}{1.01\times10^{15}}$
$= 3\times10^{-7}$ m
or 300 nm

b) $\lambda = 150$ nm

$hf = \frac{hc}{\lambda}$

$E_p = E_k + W$

$\frac{hc}{\lambda} = E_k + W$

$\frac{(4.14\times10^{-15})(3\times10^8)}{(1.5\times10^{-7})} = E_k + 4.2$

$\frac{12.42\times10^{-7}}{1.5\times10^{-7}} = E_k + 4.2$

$8.28 - 4.2 = E_k$

$\boxed{E_k = 4.08\text{ eV}}$ → $= 6.53\times10^{-19}$ J $= \frac{1}{2}mv^2$

$v = \sqrt{\frac{2(6.53\times10^{-19})}{9.11\times10^{-31}}}$
=

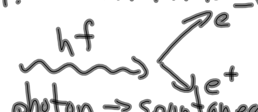
$\text{eV} \xrightarrow{\times1.6\times10^{-19}} \text{J}$

c) $V_0 = 4.08$ V

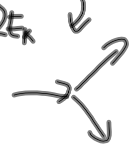
$E_k = E_e$
$6.53\times10^{-19} = qV$
$= eV$

Light-Matter Interactions

1. Reflection → most common
   → photon reflects exactly the same
2. Photoelectric Effect → photon is completely absorbed by an e^- (which is released from its atom)
3. Photon can be absorbed by an e^- which moves to a higher energy level. (A new photon can be reemitted)
4. Annihilation & pair production:



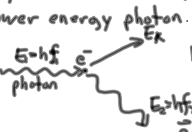
$E = 2 \times m_e c^2 + 2E_k$



photon → spontaneously turns into e^- & e^+

5. <u>Compton Effect</u>

A photon is absorbed completely by an e^-. The e^- releases a lower energy photon.



E is conserved
$hf_1 = E_k + hf_2$
but also...
$\vec{p}$ is conserved
$\vec{p} = m\vec{v}$?

$E = hf$

$E_k = \frac{1}{2}mv^2$ but $m = \frac{E}{c^2}$ ← $E = mc^2$

$p = mv$
$= \left(\frac{E}{c^2}\right)c$

$p = \frac{E}{c}$ ∴ $p = \frac{hf}{c}$ $h\left(\frac{f}{c}\right)$

or $\boxed{p = \frac{h}{\lambda}}$

Conversely, matter can have a wavelength!

$\lambda = \frac{h}{p} = \frac{h}{mv}$ ⇒ de Broglie wavelength

Eg. 1000 kg car going 90 $\frac{km}{h}$

$\lambda = \frac{h}{mv}$
$= \frac{6.63 \times 10^{-34}}{(1000)(25)}$
$= 2.65 \times 10^{-38}$ m

... with an electron going $1 \times 10^5 \frac{m}{s}$

$\lambda = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(1 \times 10^5)}$
$= 7.28 \times 10^{-9}$ m