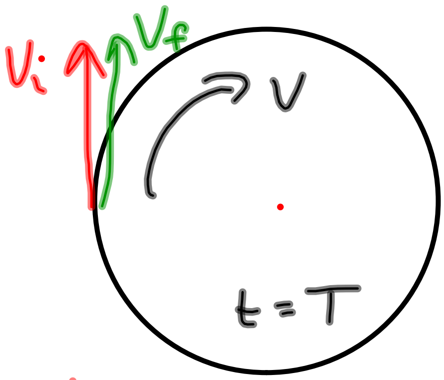
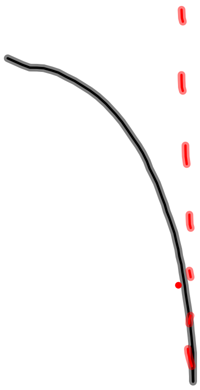
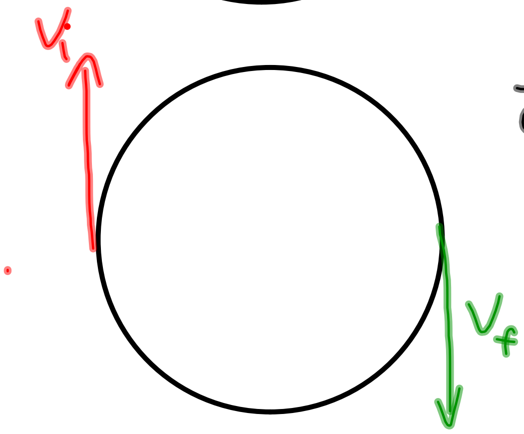
Centripetal Motion



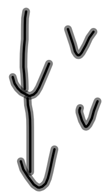
over 1 loop

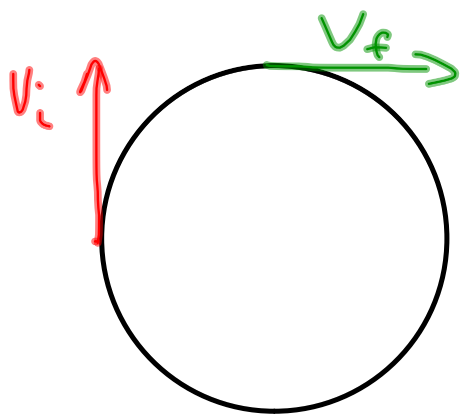
$$a_{av} = \frac{v_f - v_i}{t} = 0$$



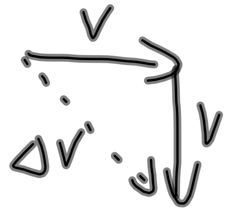


$$\vec{a}_{av} = \frac{2v}{T/2} = \frac{4v}{T}$$

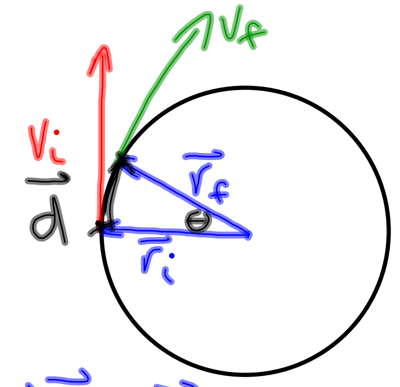




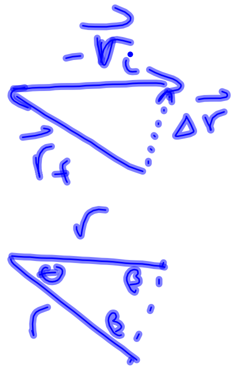
$$a = \frac{v_f - v_i}{t} = \frac{\sqrt{2}\,v}{T/4} = \frac{4\sqrt{2}\,v}{T}$$

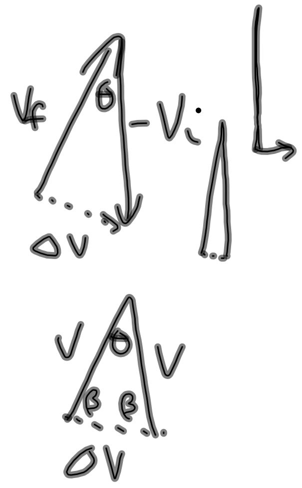


$$\Delta v^2 = v^2 + v^2$$
$$\Delta v^2 = 2v^2$$
$$\Delta v = \sqrt{2}\,v$$



$$a = \frac{v_f - v_i}{t}$$



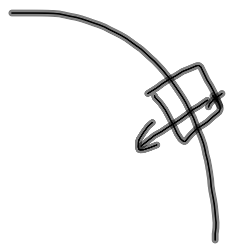


$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

Recall

$$a = \frac{\Delta v}{t}$$
$$= \frac{\Delta r\left(\frac{v}{r}\right)}{t}$$
$$= \frac{v}{r}\left(\frac{\Delta r}{t}\right)$$
$$= \frac{v}{r}\left(\frac{d}{t}\right)$$
$$= \frac{v}{r}(v)$$
$$\boxed{a = \frac{v^2}{r}}$$

$$\frac{v}{\Delta v} = \frac{r}{\Delta r}$$
(congruent Δ)
$$\Delta v = \Delta r\left(\frac{v}{r}\right)$$



$$\vec{a}_c = \frac{v^2}{r} \text{ [towards centre]}$$

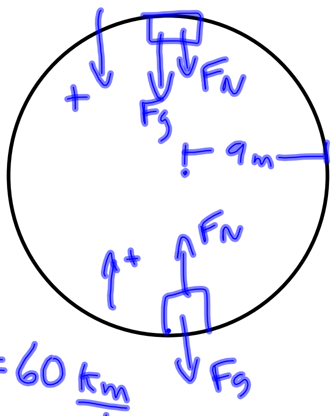
$$\vec{F}_{NET} = m\vec{a}$$
$$\vec{F}_c = \frac{mv^2}{r} \text{ [centre]}$$

$$F_{NET} = F_T$$
$$F_{NET} = \frac{mv^2}{r}$$

$$F_{NET} = \frac{mv^2}{r}$$
$$F_{NET} = F_T - F_g$$

Examples

A 70 kg person is sitting in a rollercoaster going through a loop of diameter 18 m with a speed of 60 $\frac{km}{h}$. Determine the normal force on the person at the bottom & top of the loop.



$v = 60 \frac{km}{h}$

$= 16.7 \frac{m}{s}$

Top

$F_{NET} = -F_g + F_N$

$F_{NET} = F_c = \frac{mv^2}{r}$

$\therefore \frac{mv^2}{r} = mg + F_N$

$F_N = \frac{mv^2}{r} - mg$

$= m\left(\frac{v^2}{r} - g\right)$

$= m\left(\frac{16.7^2}{9} - 9.8\right)$

$= m(21.2)$

$= 21.2m$

$= 21.2(70)$

$= 1484\ N$

Bottom

$F_{NET} = F_N - F_g$

$F_{NET} = \frac{mv^2}{r}$

$\frac{mv^2}{r} = F_N - mg$

$F_N = \frac{mv^2}{r} + mg$

$= m\left(\frac{v^2}{r} + g\right)$

$= m\left(\frac{16.7^2}{9} + 9.8\right)$

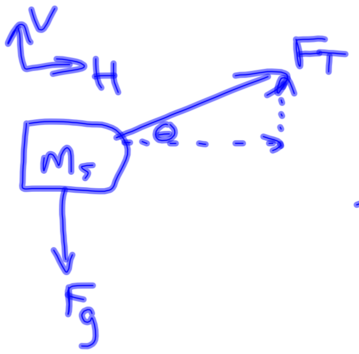
$= 40.8m$

$= 40.8(70)$

$\doteq 2856\ N$

Hor. Loop.

A stopper is twirled in a horizontal loop with a 1.0 m long string. If the frequency of rotation is 6.5 Hz, what is the tension in the string? ($m_s = 15$ g)



$F_c = \frac{mv^2}{r}$

$f = \frac{1}{T}$

for 1 loop

$t = T$

$d = 2\pi r$

$\therefore v = \frac{2\pi r}{T}$

$= 2\pi r f$

$F_c = \frac{m(2\pi r f)^2}{r}$

$= 4\pi^2 r m f^2$

or $\frac{4\pi^2 r m}{T^2}$

sub into ①

$F_T \sin 0.34 = (0.015)(9.8)$

$F_T = 25\ N$

vert.

$F_{NET} = 0$

$F_{NET} = F_T \sin\theta - F_g$

$\therefore \boxed{F_T \sin\theta = mg}$ ①

Hor.

$F_{NET} = F_c = 4\pi^2 r m f^2$

$F_{NET} = F_T \cos\theta$

$\therefore \boxed{F_T \cos\theta = 4\pi^2 r m f^2}$ ②

$\frac{①}{②} = \frac{\cancel{F_T} \sin\theta = \cancel{m} g}{\cancel{F_T} \cos\theta = 4\pi^2 r \cancel{m} f^2}$

$\tan\theta = \frac{g}{4\pi^2 r f^2}$

$f = 6.5$ Hz

$r = 1 \cos\theta$
$= \cos\theta$

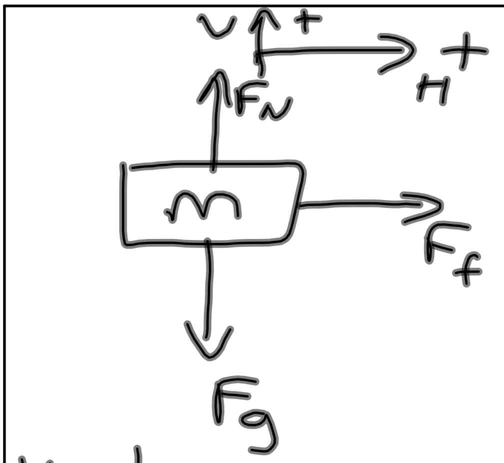
$\frac{\sin\theta}{\cancel{\cos\theta}} \cancel{\tan\theta} = \frac{g}{4\pi^2 \cancel{(\cos\theta)} f^2}$

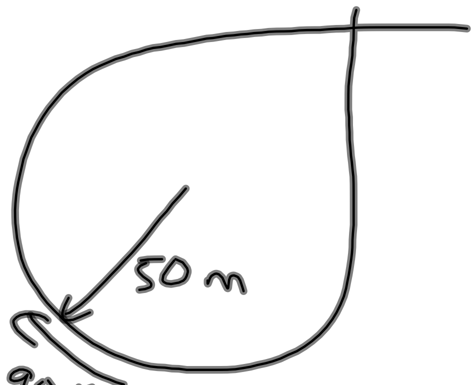
$\sin\theta = \frac{g}{4\pi^2 f^2}$

$= \frac{9.8}{1666}$

$= 5.88 \times 10^{-3}$

$\theta = 0.34°$





$90 \frac{km}{h} = 25 \frac{m}{s}$

Vert.

$F_g = F_N$

$F_N = mg$

Hor.

$F_{NET} = F_f$

$F_{NET} = F_c$

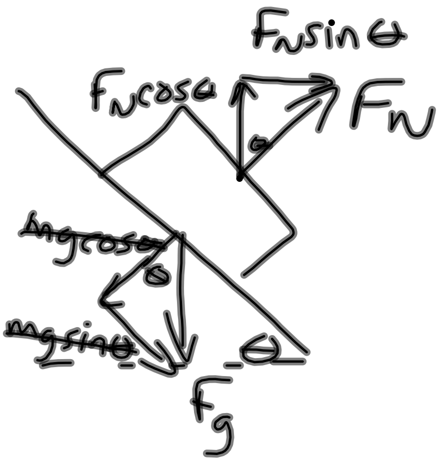
$\mu F_N = \frac{mv^2}{r}$

$\mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$

$\mu = \frac{v^2}{rg} = \frac{(25)^2}{(50)(9.8)}$

$= 1.28$

Banked Curves



Vert.

$F_{NET} = 0$

$F_{NET} = F_N \cos\theta - mg$

$\therefore F_N = \dfrac{mg}{\cos\theta} \rightarrow F_N \sin\theta = \dfrac{mv^2}{r}$

Hor.

$F_{NET} = \dfrac{mv^2}{r}$

$F_{NET} = F_N \sin\theta$

$\dfrac{\cancel{m}g}{\cos\theta} \sin\theta = \dfrac{\cancel{m}v^2}{r}$

$g \tan\theta = \dfrac{v^2}{r}$

$\tan\theta = \dfrac{v^2}{gr}$

If $v = 25 \frac{m}{s}$

$r = 50$ m

$\tan\theta = 1.28$

$\theta = 52^\circ$