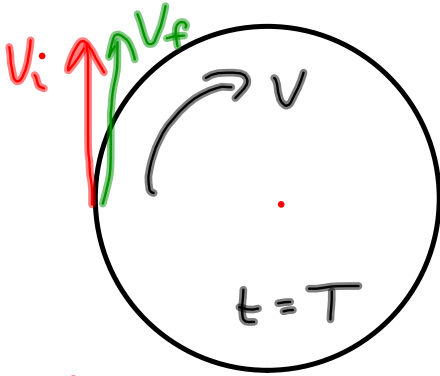


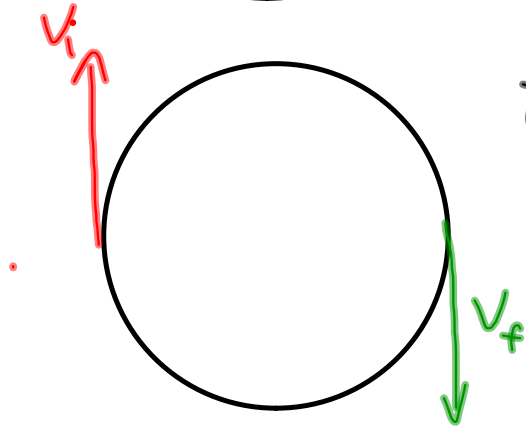
Centripetal Motion



Over 1 loop

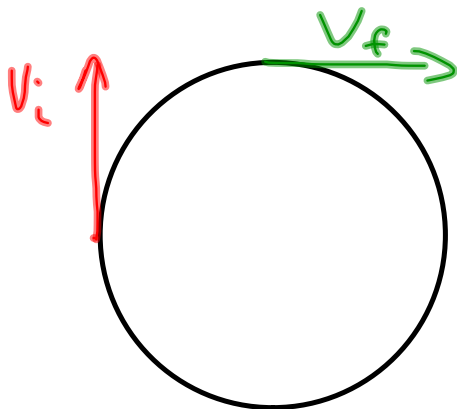
$$a_{av} = \frac{V_f - V_i}{t}$$

$$= 0$$



$$\vec{a}_{av} = \frac{2v}{T/2}$$

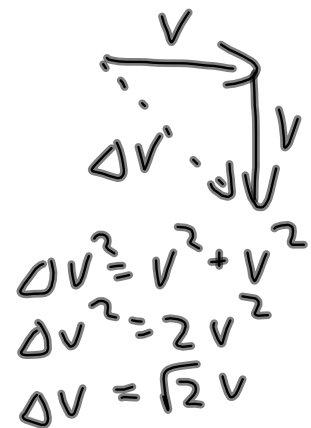
$$= \frac{4v}{T}$$

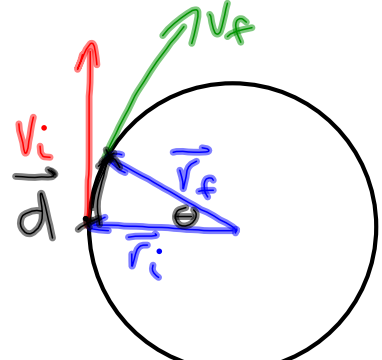


$$a = \frac{V_f - V_i}{t}$$

$$= \frac{\sqrt{2}v}{T/4}$$

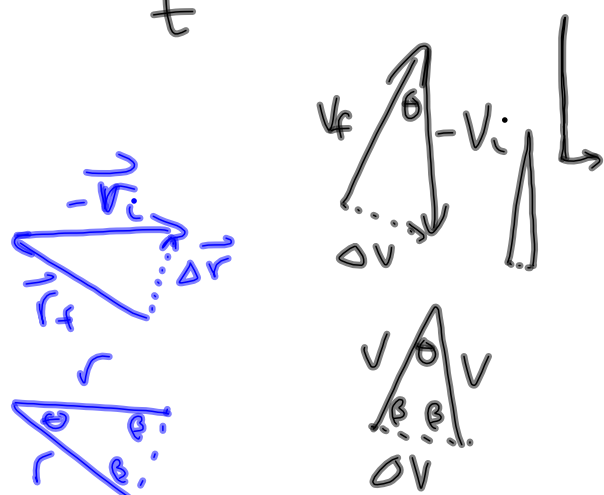
$$= \frac{4\sqrt{2}v}{T}$$





$a = \frac{v_f - v_i}{t}$

$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$



Recall

$$a = \frac{\Delta v}{t}$$

$$= \frac{\Delta r \left(\frac{v}{r}\right)}{t}$$


$$= \frac{v}{r} \left(\frac{\Delta r}{t}\right)$$

$$= \frac{v}{r} \left(\frac{d}{t}\right)$$

$$= \frac{v}{r} (v)$$

$$a = \frac{v^2}{r}$$

$\vec{a}_c = \frac{v^2}{r}$ [towards centre]



$F_{NET} = m\vec{a}$

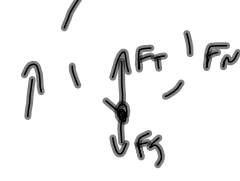
$F_{NET} = \frac{mv^2}{r}$ [centre]

$F_{NET} = F_T$

$F_{NET} = \frac{mv^2}{r}$

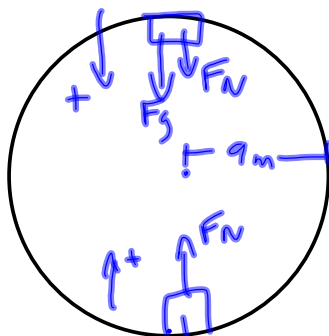
$F_{NET} = \frac{mv^2}{r}$

$F_{NET} = F_T - F_g$



Examples

A 70 kg person is sitting in a rollercoaster going through a loop of diameter 18 m with a speed of $60 \frac{\text{km}}{\text{h}}$. Determine the normal force on the person at the bottom & top of the loop.



$$v = 60 \frac{\text{km}}{\text{h}}$$

$$= 16.7 \frac{\text{m}}{\text{s}}$$

Bottom

$$F_{\text{NET}} = F_N - F_g$$

$$F_{\text{NET}} = \frac{mv^2}{r}$$

$$\frac{mv^2}{r} = F_N - mg$$

$$F_N = \frac{mv^2}{r} + mg$$

$$= m \left(\frac{v^2}{r} + g \right)$$

$$= m \left(\frac{16.7^2}{9} + 9.8 \right)$$

$$= 40.8 m$$

$$= 40.8(70)$$

$$= 2856 \text{ N}$$

TOP $F_{\text{NET}} = F_g + F_N$

$$F_{\text{NET}} = F_c = \frac{mv^2}{r}$$

$$\therefore \frac{mv^2}{r} = mg + F_N$$

$$F_N = \frac{mv^2}{r} - mg$$

$$= m \left(\frac{v^2}{r} - g \right)$$

$$= m \left(\frac{16.7^2}{9} - 9.8 \right)$$

$$= m(21.2)$$

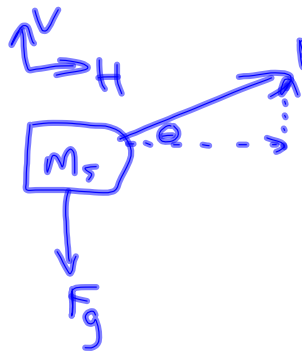
$$= 21.2 m$$

$$= 21.2(70)$$

$$= 1484 \text{ N}$$

Hor. Loop.

A stopper is twirled in a horizontal loop with a 1.0 m long string. If the frequency of rotation is 6.5 Hz, what is the tension in the string? ($m_s = 15 \text{ g}$)



vert.

$$F_{NET} = 0$$

$$F_{NET} = F_T \sin \theta - F_g$$

$$\therefore F_T \sin \theta = mg \quad (1)$$

Hor.

$$F_{NET} = F_c = 4\pi^2 r m f^2$$

$$F_{NET} = F_T \cos \theta$$

$$\therefore F_T \cos \theta = 4\pi^2 r m f^2 \quad (2)$$

$$\frac{(1)}{(2)} = \frac{F_T \sin \theta = mg}{F_T \cos \theta = 4\pi^2 r m f^2}$$

$$\tan \theta = \frac{g}{4\pi^2 r f^2}$$

$$f = 6.5 \text{ Hz}$$



$$\frac{\sin \theta}{\cos \theta} \tan \theta = \frac{g}{4\pi^2 (\cos \theta) f^2}$$

$$\sin \theta = \frac{g}{4\pi^2 f^2}$$

$$= \frac{9.8}{1666}$$

$$= 5.88 \times 10^{-3}$$

$$\theta = 0.34^\circ$$

$$F_c = \frac{mv^2}{r}$$

$$f = \frac{1}{T}$$

for 1 loop

$$t = T$$

$$d = 2\pi r$$

$$\therefore v = \frac{2\pi r}{T}$$

$$= 2\pi r f$$

$$F_c = \frac{m(2\pi r f)^2}{r}$$

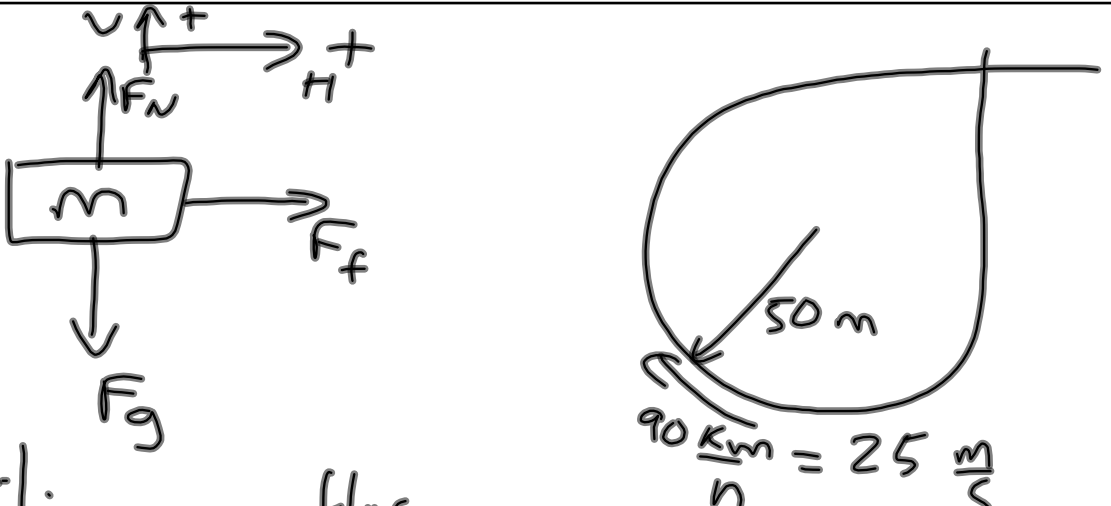
$$= 4\pi^2 r m f^2$$

$$\text{or } \frac{4\pi^2 r m}{T^2}$$

sub into (1)

$$F_T \sin 0.34 = (0.015)(9.8)$$

$$F_T = 25 \text{ N}$$



The image contains two diagrams. On the left is a free-body diagram of a mass m in a rectangular box. Four force vectors are shown: F_N pointing vertically upwards, F_g pointing vertically downwards, F_f pointing horizontally to the right, and H^+ pointing horizontally to the right. Above the box, there are two coordinate axes: a vertical axis labeled v with an upward arrow and a horizontal axis labeled H^+ with a rightward arrow. On the right is a diagram of a circular arc representing a banked curve. The radius of the arc is labeled as 50 m . A curved arrow indicates the direction of motion, and the speed is given as $90 \frac{\text{km}}{\text{h}} = 25 \frac{\text{m}}{\text{s}}$.

Verl.

$$F_g = F_N$$

$$F_N = mg$$

Hor.

$$F_{N\text{ET}} = F_f$$

$$F_{N\text{ET}} = F_c$$

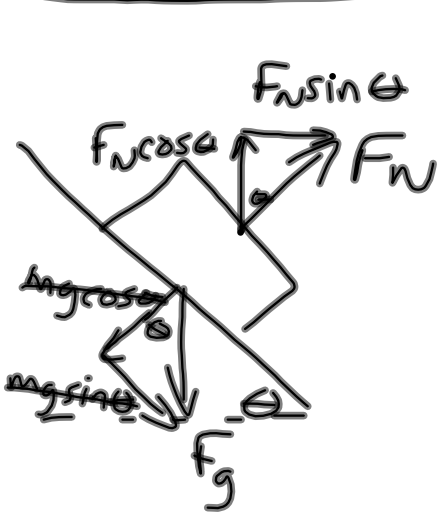
$$\mu F_N = \frac{mv^2}{r}$$

$$\mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{rg} = \frac{(25)^2}{(50)(9.8)}$$

$$= 1.28$$

Banked Curves



vert.

$$F_{NET} = 0$$

$$F_{NET} = F_N \cos \theta - mg$$

$$\therefore F_N = \frac{mg}{\cos \theta}$$

Hor.

$$F_{NET} = \frac{mv^2}{r}$$

$$F_{NET} = F_N \sin \theta$$

$$\therefore F_N \sin \theta = \frac{mv^2}{r}$$

If $v = 25 \frac{m}{s}$
 $r = 50 \text{ m}$

$$\tan \theta = 1.28$$

$$\theta = 52^\circ$$

$$\frac{mg \sin \theta}{\cos \theta} = \frac{mv^2}{r}$$

$$g \tan \theta = \frac{v^2}{r}$$

$$\tan \theta = \frac{v^2}{gr}$$