Impulse & Momentum

$\vec{p} = m\vec{v}$

↳ unit: $\frac{kg\,m}{s}$

$5\ \frac{kg\,m}{s}$

$\frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta m\vec{v}}{\Delta t}$

$= \frac{m\Delta v}{\Delta t}$

$= m\frac{\Delta v}{\Delta t}$

$= ma$

$= F$

Impulse, $\vec{J}$

↳ $\Delta \vec{p}$

$\frac{\Delta \vec{p}}{\Delta t} = \vec{F}$

$\therefore \Delta \vec{p} = \vec{F}\Delta t$

N^{+}

Example:

A 152 g baseball moving $150\ \frac{km}{h}$ [S] is hit by a bat to propel it to $200\ \frac{km}{h}$ [N] in 0.015 s. What is the impulse on the ball?

$v_i = -41.7\ \frac{m}{s}$

$v_f = 55.6\ \frac{m}{s}$

$m = 0.152\ kg$

$\vec{J} = \Delta \vec{p} = m\Delta \vec{v}$

$= (0.152)(v_f - v_i)$

$= 0.152(55.6 - (-41.7))$

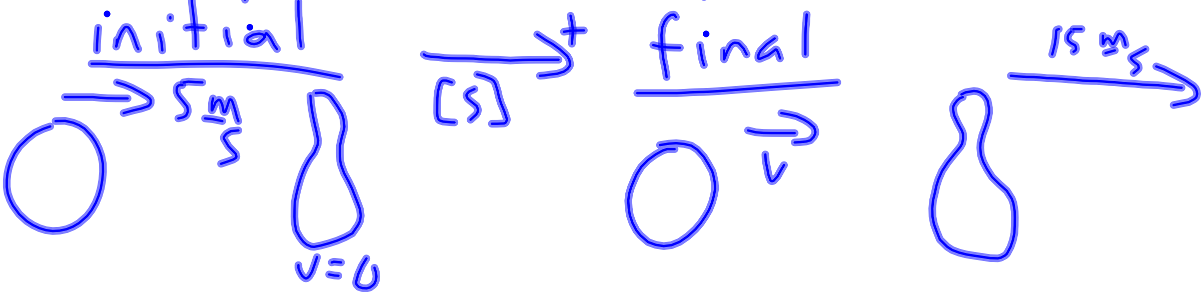
$= 14.8\ kg\frac{m}{s}$

Conservation of Momentum

→ momentum stays the same in a system

1-D

A 16 kg bowling ball moving south at 5 $\frac{m}{s}$ strikes a stationary 1.6 kg pin. After the collision the pin flies off at 15 $\frac{m}{s}$ [S]. What is the new velocity of the ball?



ball

$m = 16$ kg

$V_i = 5\ \frac{m}{s}$

$V_f = V$

pin

$m = 1.6$ kg

$V_i = 0$

$V_f = 15\ \frac{m}{s}$

$$\vec{p_i} = \vec{p_f}$$

$$m_b V_{i_b} + m_p V_{i_p} = m_b V_{f_b} + m_p V_{f_p}$$

$$(16)(5) + 0 = 16v + 1.6(15)$$

$$80 = 16v + 24$$

$$80 - 24 = 16v$$

$$56 = 16v$$

$$v = 3.5\ \frac{m}{s}$$

Elastic Collisions

In an elastic collision, 100% of the kinetic energy is conserved, but only when the collision is done.

Eg. → bowling question
but we don't know any v_f's
and the collision is elastic

$\xrightarrow{\;\;}$ S+

Ball	Pin
$m = 16\ kg$	$m = 1.6\ kg$
$v_i = 5\ \frac{m}{s}$	$v_i = 0\ \frac{m}{s}$
$v_f = v_B$	$v_f = v_p$

$$\vec{p}_i = \vec{p}_f$$
$$\vec{p}_{iB} + \cancel{\vec{p}_{ip}}^{\,v_{ip}=0} = \vec{p}_{fB} + \vec{p}_{fp}$$
$$(16)(5) = (16)v_B + (1.6)v_p$$
$$\boxed{80 = 16v_B + 1.6v_p}$$
$$80 - 16v_B = 1.6v_p$$
$$v_p = \frac{80 - 16v_B}{1.6}$$
$$\boxed{v_p = 50 - 10v_B}$$

0.9 $E_{k_i} = E_{k_f}$

$$E_{k_{i_B}} + \cancel{E_{k_{i_p}}}^{\,v=0} = E_{k_{f_B}} + E_{k_{f_p}}$$
$$\frac{1}{2}(16)(5)^2 = \frac{1}{2}(16)v_B^2 + \frac{1}{2}(1.6)v_p^2$$
$$\boxed{200 = 8v_B^2 + 0.8v_p^2}$$

$$200 = 8v_B^2 + 0.8(50^2 - 2(50)(10v_B) + 100v_B^2)$$
$$200 = 8v_B^2 + 0.8(2500 - 1000v_B + 100v_B^2)$$
$$200 = 8v_B^2 + 2000 - 800v_B + 80v_B^2$$
$$0 = 88v_B^2 - 800v_B + 1800 \quad \div 8$$
$$0 = 11v_B^2 - 100v_B + 225$$

$$v_B = \frac{100 \pm \sqrt{10000 - 4(11)(225)}}{2(11)}$$
$$\frac{-100 \pm \sqrt{10000 - 9900}}{22}$$
$$= \frac{100 \pm 10}{22} \quad \therefore v_B = 5\ m/s \text{ or } v_B = 4.1\ m/s$$

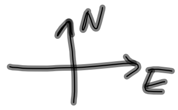
$v_p = 0$

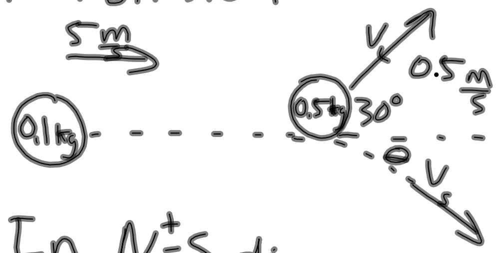
$v_p = 9\ \frac{m}{s}$

2-D Collisions

Conservation of momentum applies to 2-D collisions as well.

Y- collision





$E_{K_i} = \frac{1}{2}(0.1)(5)^2$

$= 1.25\ J$

$E_{K_f} = \frac{1}{2}(0.1)(3.1)^2 + \frac{1}{2}(0.5)(0.5)^2$

$= 0.543\ J$

In N$\pm$S dir

$\vec{P}_i = 0$

$\vec{P}_f = \vec{P}_{f_L} + \vec{P}_{f_s}$

$= (0.5)(0.5)\sin 30 - (0.1)V_s \sin\theta$

$0 = 0.125 - 0.1V_s \sin\theta$

$0.1 V_s \sin\theta = 0.125$

$\boxed{V_s \sin\theta = 1.25}$ (1)

E-W

$P_i = P_f$

$(0.1)(5) = (0.5)(0.5)\cos 30 + 0.1V_s\cos\theta$

$0.5 = 0.2165 + 0.1\, V_s\cos\theta$

$0.2835 = 0.1\, V_s\cos\theta$

$\boxed{2.835 = V_s\cos\theta}$ (2)

$\frac{(1)}{(2)}$ $\frac{1.25}{2.835} = \frac{\cancel{V_s}\sin\theta}{\cancel{V_s}\cos\theta}$

$\tan\theta = \frac{1.25}{2.835}$

$\theta = 24°$ or $\cancel{204}$

sub into (1) or (2)

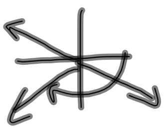
$V_s\cos 24 = 2.835$

$V_s = 3.1\ \frac{m}{s}$

$\therefore \vec{V}_s = 3.1\frac{m}{s}$ [E 24° S]

$V_s\cos 204 = 2.835$

$V_s = -3.1\frac{m}{s}$



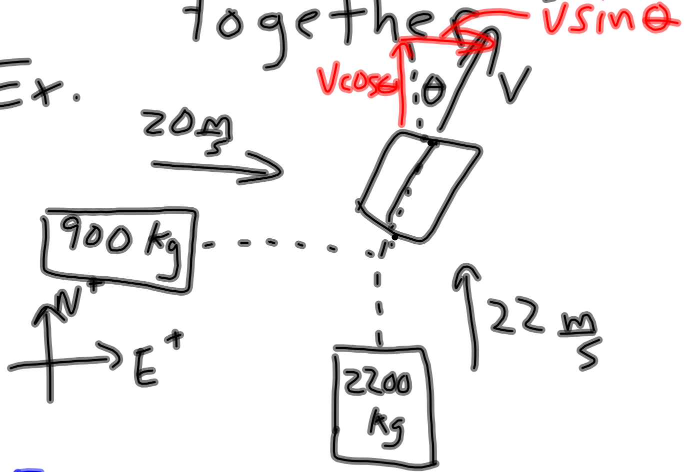
Perfectly Inelastic Colisions

↳ E_k is minimized (max. E_k lost)

↳ coliding objects stick together

$\vec{p}_i = \vec{p}_f$

Ex.



N-S

$p_i = p_f$

$(2200)(22) = (2200+900)\,v\cos\theta$

$48400 = 3100\,v\cos\theta$

$\boxed{15.6 = v\cos\theta}$

E-W

$p_i = p_f$

$(900)(20) = (3100)\,v\sin\theta$

$18\,000 = 3100\,v\sin\theta$

$\boxed{5.8 = v\sin\theta}$

$\frac{5.8}{15.6} = \frac{\cancel{v}\sin\theta}{\cancel{v}\cos\theta}$

$\tan\theta = 0.372$

$\theta = 20^\circ$

$5.8 = v\sin 20$

$v = 17\ \frac{m}{s}$

$\therefore \vec{v} = 17\ \frac{m}{s}\ [N\ 20^\circ\ E]$

$E_{K_i} = \frac{1}{2}(900)(20)^2 + \frac{1}{2}(2200)(22)^2$

$= 180\,000 + 532\,400$

$= 712\,400\ J$

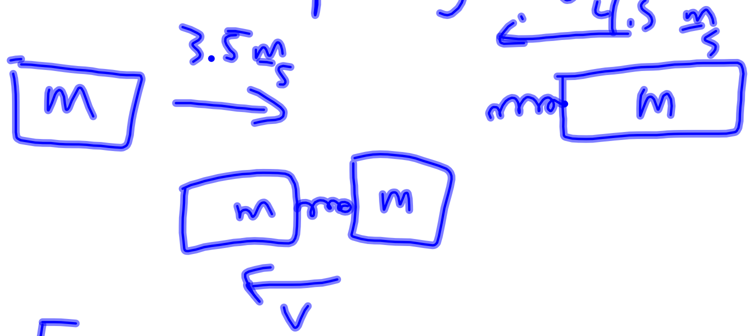
$E_{K_f} = \frac{1}{2}(3100)(17)^2$

$= 447\,950\ J$

$\Delta E = -264\,000\ J$

Ex. 2

Two carts of mass 2 kg are moving towards each other at velocities of 3.5 $\frac{m}{s}$ (E) and 4.5 $\frac{m}{s}$ [W]. A spring ($k = 900 \frac{N}{m}$) is attached to one of them. What is the maximum compression of the spring?



$p_i = p_f$

$m(3.5) + m(-4.5) = (m+m)v$

$-\cancel{m} = 2\cancel{m}v$

$v = -0.5 \frac{m}{s}$

$E_{k_i} = \frac{1}{2}(2)(3.5)^2 + \frac{1}{2}(2)(4.5)^2$

$= 32.5\ J$

$E_{k_f} = \frac{1}{2}(4)(0.5)^2$

$= 0.5\ J$

$\Delta E = -32\ J$

$\therefore$ 32 J goes into the spring

$\therefore E_s = 32\ J$

$k = 900 \frac{N}{m}$

$E_s = \frac{1}{2}kx^2$

$x = \sqrt{\frac{2(32)}{900}}$

$= 0.267\ m$ or 26.7 cm