Unit 2: Energy & Momentum

Work-Energy Theorem:

Energy is the ability to do work
Work is the transfer of (or change in) energy.

$W = \Delta E$ (in joules, J)

$W = \vec{F} \cdot \vec{d}$
$= |F||d|\cos\theta$
$= Fd\cos\theta$

zero work → when $\vec{d} = 0$
→ when $\vec{F} = 0$
→ when $\vec{F} \perp d$

Kinetic Energy

$W = \vec{F} \cdot \vec{d}$
$= Fd$
but $F = ma$
$\therefore W = mad$
$\therefore W = m\left(\frac{1}{2}v^2\right)$
So $\Delta E_k = \frac{1}{2}mv^2$
but $E_{ki} = 0$
$\therefore$ $E_k = \frac{1}{2}mv^2$

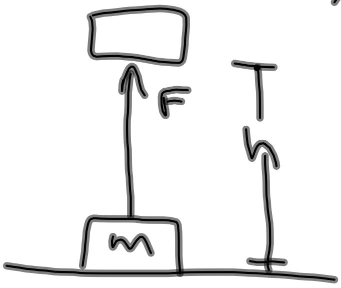
$v_f^2 = v_i^2 + 2ad$
in this case $v_i = 0$
$v_f = v$
$\therefore v^2 = 2ad$
$\frac{1}{2}v^2 = ad$

$\Delta E_K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$[J] = [kg]\left[\frac{m}{s}\right]^2$

$1\,J = 1\,\frac{kg\,m^2}{s^2}$

Gravitational Energy (on Earth's surface)



$h = 0$

$$W = \vec{F} \cdot \vec{d}$$

$$W = \Delta E_g \quad \text{but } E_{g_i} = 0$$

$$\therefore W = E_g$$

$$\therefore F_g h = E_g$$

so $\boxed{E_g = mgh}$

$$[J] = [kg]\left[\frac{m}{s^2}\right][m]$$

$$[J] = \frac{kg\,m^2}{s^2}$$

Conservation of Mechanical Energy

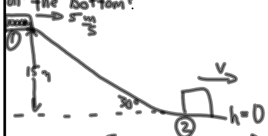
Mechanical Energy → the sum of E_K and E_g

When no work is done on an object other than gravity, the mechanical energy remains constant.

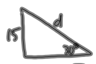
If work is done...

$$W = \Delta E$$
$$= E_f - E_i$$
$$\vec{F} \cdot \vec{d} = (E_{g_f} + E_{K_f}) - (E_{g_i} + E_{K_i})$$

A rollercoaster ($m = 1000$ kg) is moving at 5 m/s at the top of a 15 m high hill. The hill makes an angle of 30° to the ground. If someone straps a rocket engine to the coaster to apply a 5000 N force, how fast is it moving at the bottom?



$$E_1 = E_{K_1} + E_{g_1}$$
$$= \tfrac{1}{2} m v_1^2 + mgh_1$$
$$= \tfrac{1}{2}(100)(5)^2 + (1000)(9.8)(15)$$
$$= 159\,500 \text{ J}$$



$$\sin 30 = \frac{15}{d}$$
$$d = \frac{15}{\sin 30}$$
$$= 30 \text{ m}$$

$$E_2 = \tfrac{1}{2} m v_2^2 + mgh_2$$
$$= 500 v^2$$

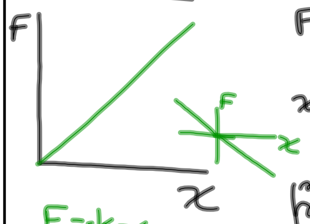
$$W = \Delta E$$
$$= E_2 - E_1$$
$$= 500 v^2 - 159\,500$$

and $W = \vec{F} \cdot \vec{d}$ $(\theta = 0°)$
$$= (5000)(30)$$
$$= 150\,000 \text{ J}$$

$$150\,000 = 500 v^2 - 159\,500$$
$$309\,500 = 500 v^2$$
$$v = 25 \frac{m}{s}$$

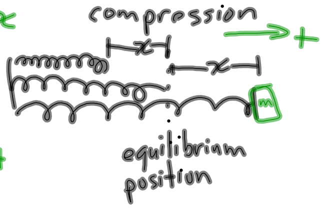
$$v_f^2 = v_i^2 + 2ad$$
$$(25)^2 = (5)^2 + 2a(30)$$
$$a = 10 \frac{m}{s^2}$$

Springs



F - force exerted on (or by) a spring

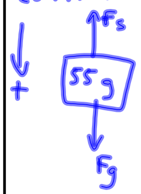
x - stretch or compression



$F = -kx$

k → spring constant $\left(\frac{N}{m}\right)$ or $\left(\frac{N}{cm}\right)$

A spring is 10 cm long when hanging vertically. When a 55 g mass is attached it stretches to 25 cm. What is the spring constant?



$F_{NET} = 0$

$F_{NET} = F_g - F_s$

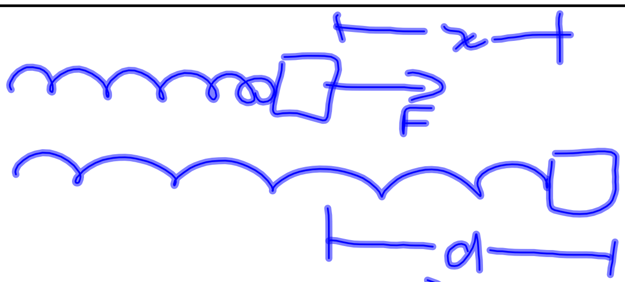
$\therefore F_g = F_s$

$mg = kx \rightarrow k = \frac{mg}{x}$

$x = 25 - 10$
$= 15\text{ cm}$
$= 0.15\text{ m}$

$= \frac{(0.055)(9.8)}{0.15}$

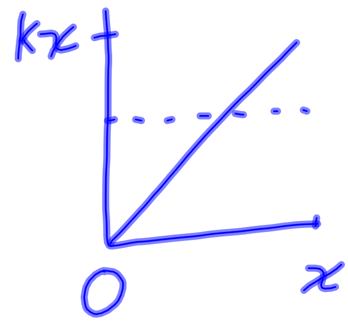
$= 3.6\ \frac{N}{m}$



$W = E_s = \vec{F_s} \cdot \vec{d}$

$= -\left(-\frac{1}{2}kx\right)(x)$

$E_s = \frac{1}{2}kx^2$



$\frac{0 + kx}{2}$