Orbital Physics

Recall...

$F_g = \frac{G m_1 m_2}{r^2}$ where m_1 & m_2 are the masses attracting each other

r is the distance between them

and G is the universal gravitational constant $= 6.67 \times 10^{-11} \frac{N m^2}{kg^2}$

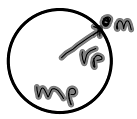
On the surface of a celestial body

$F_g = mg$ $g \rightarrow$ depend on the body

$\downarrow$ gravitation field constant

$g = \frac{F_g}{m}$ $\left[\frac{N}{kg}\right]$

$mg = \frac{G m_1 m_2}{r^2}$

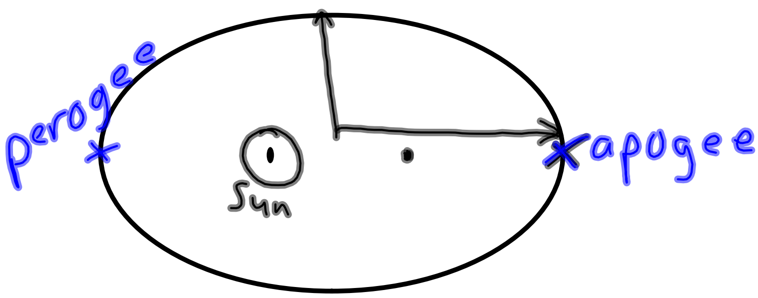


$\cancel{m}g = \frac{G m_p \cancel{m}}{r_p^2}$

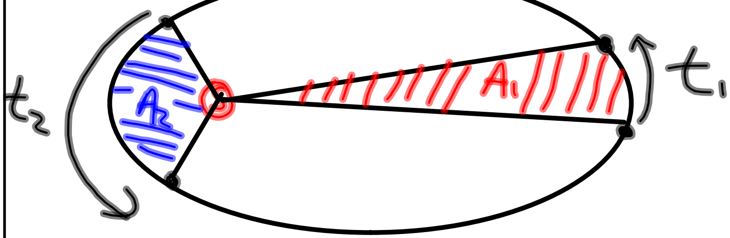
$g = \frac{G m_p}{r_p^2}$

Kepler's Laws of Planetary Motion

1. Each planet moves around the sun in an ellipse, with the sun at one focus.

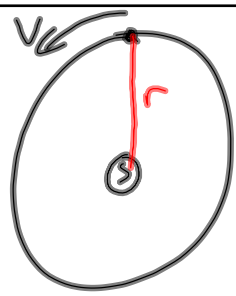


2. The straight line joining the planet and the sun will sweep out equal areas in equal times.



If $t_1 = t_2$
then $A_1 = A_2$

3.



$$F_{NET} = F_c = \frac{4\pi^2 r m_p}{T^2}$$

$$F_{NET} = F_g = \frac{G m_s m_p}{r^2}$$

$$\frac{G \cancel{m_p} m_s}{r^2} = \frac{4\pi^2 r \cancel{m_p}}{T^2}$$

$$G m_s T^2 = 4\pi^2 r^3$$

$$T^2 = \frac{4\pi^2}{G \underline{m_s}} r^3$$

* mass being orbited

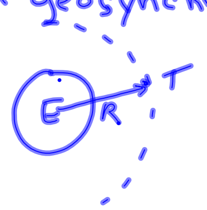
The cube of the average orbital radius is proportional to the square of the orbital period.

$$R^3 = KT^2 \qquad \frac{R^3}{T^2} = K$$

for all orbitting objects going around the same thing

K3L Example

Determine the orbital radius of a geosynchronous satellite.



$F_c = F_g$

$\frac{4\pi^2 r \cancel{m}}{T^2} = \frac{G\cancel{m} m_E}{r^2}$

$m_E = 5.98\times10^{24}\ kg$

$T = 1\ day$
$= 24\times60\times60$
$= 86\,400\ s$

$R^3 = \frac{Gm_E}{4\pi^2} T^2$

$= \frac{(6.67\times10^{-11})(5.98\times10^{24})(86400)^2}{4\pi^2}$

$K_E = \frac{Gm_E}{4\pi^2}$
$= 1.01\times10^{13}\ \frac{m^3}{s^2}$

$R^3 = 7.48\times10^{22}\ m$

$R = 4.2\times10^{7}\ m$

$\sim 42\,000\ km$
$- 6\,400\ km\ (r_E)$

$35\,600\ km$ above surface

method 2

$R^3 = K_E T^2$

$\frac{R_s^3}{T_s^2} = \frac{R_m^3}{T_m^2}$

Satellite Moon

$T_s = 24\ h$

$T_m = 27\ days\ 8\ h$
$= 656\ h$

$R_m = 384\,000\ km$

$R_s^3 = \frac{R_m^3 T_s^2}{T_m^2}$

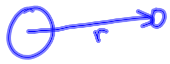
$R_s = 42\,000\ km$

Gravitational Energy

$E_g = F_g d$

$= \frac{Gm_1m_2}{r^2} \cdot r$

$= -\frac{Gm_1m_2}{r}$



$E_g = 0$ at ∞

As $r\downarrow$ $E_g\downarrow$

How much gravitational energy does a satellite have when it is 20 000 km above Earth's surface?

$m_s = 3000$ kg

$r = 20\,000$ km $+ r_E$

$= 26\,400$ km

$= 2.64 \times 10^7$ m

$m_E = 5.98 \times 10^{24}$ kg

$E_k = ?$

$E_g = -\frac{Gm_1m_2}{r}$

$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(3000)}{2.64\times10^7}$

$= -4.53 \times 10^{10}$ J

$F_c = F_g$

$\frac{\cancel{m_s}v^2}{\cancel{r}} = \frac{Gm_E\cancel{m_s}}{r^{\cancel{2}}}$

$v^2 = \frac{Gm_E}{r}$

$E_K = \frac{1}{2}m_sv^2$

$= \frac{1}{2}m_s\left(\frac{Gm_E}{r}\right)$

$= \frac{1}{2}\frac{Gm_Em_s}{r}$

$= -\frac{1}{2}E_g$

$= -\frac{1}{2}(-4.53\times10^{10})$

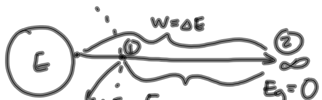
$= 2.26\times10^{10}$ J

$E_T = E_g + E_K$

$= E_g + (-\frac{1}{2}E_g)$

$= \frac{1}{2}E_g$

Binding Energy



$E_g = 0$

$E_k = 0$

$E_T = 0$

$E_T = \frac{1}{2}E_g$ E_B

Binding Energy

from yesterday

$E_1 = \frac{1}{2}E_g$

$= \frac{1}{2}\left(\frac{-Gmm_E}{r}\right)$

$E_2 = 0$

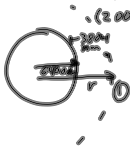
$E_B = E_2 - E_1$

$= 0 - \frac{1}{2}E_g$

$= \frac{1}{2}\left(\frac{Gmm_E}{r}\right) = -\frac{1}{2}E_g$

What is the binding energy of a satellite 380 km above Earth's surface?

(2000 kg)



$r = 6380 + 380$

$= 6760$ km

$= 6.76\times10^6$ m

$E_1 = E_{g_1} + E_{k_1}$

$E_{g_1} = -\frac{Gm_sm_E}{r}$

$= -\frac{(6.67\times10^{-11})(2000)(5.98\times10^{24})}{6.76\times10^6}$

$= -1.18\times10^{11}$ J

$F_c = F_g$

$\frac{Gm_sm_E}{r^2} = \frac{m_sv^2}{r}$

$v = \sqrt{\frac{Gm_E}{r}}$

$= \sqrt{\frac{3.95\times10^{14}}{6.76\times10^6}}$

$= 7.64\times10^3$ m/s

$E_k = \frac{1}{2}m_sv^2$

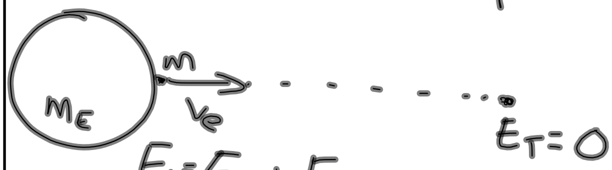
$= \frac{1}{2}(2000)(7.64\times10^3)^2$

$= 5.84\times10^{10}$ J

$E_T = -1.18\times10^{11} + 5.84\times10^{10}$

$= -5.96\times10^{10}$ J

Escape Speed $\rightarrow$ speed required to be launched at to leave the planet's gravity



$E_1 = E_K + E_g$

G ✓
$m_1 = m_E$
$m_2 = m$
$r = r_E$
$v = v_e$

$$E_K + E_g = 0$$
$$E_{K_2} = -E_g$$
$$\frac{1}{2}\cancel{m}v_e^2 = -\left(-\frac{G m_E \cancel{m}}{r_E}\right)$$
$$v_e^2 = \frac{2Gm_E}{r_E}$$
$$v_e = \sqrt{\frac{2Gm_E}{r_E}}$$
$$= 11\,000\ \frac{m}{s} \text{ or } 11\ \frac{km}{s}$$