Rate of Radioactive Decay

Whether an individual nucleus of an isotope decays at any given time is a mostly random event.
However, when a large number of nuclei are considered, there is a statistical pattern .

Consider the rolling of a 6-sided dice. Suppose rolling a 1 means that the nucleus decays. After one roll, there is a 1 in 6 chance that the nucleus decays. If it survives, there is a 1 in 6 chance it decays on the next roll. This pattern continues indefinitely. The nucleus, may NEVER decay (although this statistically impossible given enough rolls). Notice that the decay IS random, although the randomness is governed by statistic.

Decay (continued...)

Now suppose we have 60 dice. We would expect about 10 of them to decay on the first roll. However, we could have all of them decay, or none of them decay!

Let's try it ourselves...

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We likely notice that the pattern we expected did not work perfectly. With such a low number, the likelihood of getting a perfectly statistical pattern is very small.

Let's see what would happen if we had ~~6~~40 000 000 dice instead...

Decay (continued...)

Notice that in the graph with the larger number of samples it takes approximately the same amount of time for the total amount of parent isotope to be cut in half regardless of where you start from. Let's call this the half-life. $t_{1/2}$

Half-life: The time it takes for half of the nuclei of a radioactive isotope to decay.

Let's refer to this as $t_{1/2}$

Also, let the number of original parent isotope be N_0,
the number of parent isotope at some time, t, be N
and the number of half-lives that have passed be n

Half-life Calculations

N_0

N

n = number of half-lives → $\boxed{\dfrac{t}{t_{1/2}}} = n$

$$N_0 \xrightarrow{t_{1/2}} \frac{1}{2}N_0 \xrightarrow{t_{1/2}} \frac{1}{4}N_0 \xrightarrow{t_{1/2}} \frac{1}{8}N_0 \cdots$$

$$\boxed{N = N_0\left(\frac{1}{2}\right)^n}$$

How long does it take for only 10% of a U-238 sample to remain

$t_{1/2} = 4.5 \times 10^9$ years

$t =$

$n =$

$N = \frac{1}{10} N_0$ or $\frac{N}{N_0} = \frac{1}{10}$

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$0.1 = \left(\frac{1}{2}\right)^n$$

$$\log 0.1 = \log 0.5^n$$

$$\log 0.1 = n \log 0.5$$

$$\frac{\log 0.1}{\log 0.5} = n$$

$$n = 3.32$$

$t = n \times t_{1/2}$

$= 15$ billion years

Radiometric Dating

When doing radiometric dating, we compare the parent isotope to the daughter isotope.

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$D = N_0 - N$$

$$= N_0 - N_0\left(\frac{1}{2}\right)^n$$

$$= N_0\left(1 - \left(\frac{1}{2}\right)^n\right)$$

$$\frac{D}{N} = \frac{\cancel{N_0}\left(1-\left(\frac{1}{2}\right)^n\right)}{\cancel{N_0}\left(\frac{1}{2}\right)^n}$$

$$= \frac{1-\left(\frac{1}{2}\right)^n}{\left(\frac{1}{2}\right)^n}$$

$$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

$$\therefore \frac{1}{\left(\frac{1}{2}\right)^n} - \frac{\left(\frac{1}{2}\right)^n}{\left(\frac{1}{2}\right)^n}$$

$$= \frac{1}{\frac{1}{2^n}} - 1$$

$$\boxed{\frac{D}{N} = 2^n - 1}$$

Isotope	Half-life (years)
U-238	4.5×10^9
Rb-87	4.7×10^{10}
K-40	1.3×10^9
C-14	5730

1. What % of a radioisotope is left after…
i) 1 half-life
ii) 4 half-lives
iii) 0.5 half-lives
iv) 10 half-lives
v) 4.23 half-lives

2. How many half-lives have passes if..
i) 50% of a radioisotope remains
ii) 10% of a radioisotope remains
iii) 75% of a radioisotope remains
iv) $1/16^{th}$ of a radioisotope remains
v) $1/6^{th}$ of a radioisotope remains

3. Uranium-238 decays into lead-206. How much time has passed if the ratio of lead-206 to uranium-238 is...
i) 1:1
ii) 31:1
iii) 12:1
iv) 1:5

4. In order to date a fossil found buried deep in a layer of sediment, the surrounding rocks need to be dated. A particular r is rich in potassium, so radiometric dating using K-40 is possible. In the surrounding rocks there is 12.1 times more K-40 than i daughter isotope, Ar-40. Determine the age of the rock, and thus, the fossil.

5. What assumptions must be made in order to accept the answer from 4? Are there any ways to prove that the assumptio are correct?

6. A wooden bowl found in an ancient Egyptian village contains 40% of the amount of C-14 that there would have been in air around that time. Determine the age of the bowl.

7. What assumptions must be made in order to accept the answer from 4? Are there any ways to prove that the assumptio are correct?

Isotope	Half-life (years)
U-238	4.5×10^9
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K-40	1.3×10^9
C-14	5730

1. What % of a radioisotope is left after…
i) 1 half-life
ii) 4 half-lives
iii) 0.5 half-lives
iv) 10 half-lives
v) 4.23 half-lives

i) 50
ii) 6.25
iii) 70.7
iv) 0.098
v) 5.3

2. How many half-lives have passes if..
i) 50% of a radioisotope remains
ii) 10% of a radioisotope remains
iii) 75% of a radioisotope remains
iv) 1/16th of a radioisotope remains
v) 1/6th of a radioisotope remains

i) 1
ii) 3.32
iii) 0.42
iv) 4
v) 2.58

3. Uranium-238 decays into lead-206. How much time has passed if the ratio of lead-206 to uranium-238 is...
i) 1:1
ii) 31:1
iii) 12:1
iv) 1:5

i) 4 500 000 000 years
ii) 22 500 000 000 years
iii) 16 700 000 000 years
iv) 1 180 000 000 years

4. In order to date a fossil found buried deep in a layer of sediment, the surrounding rocks need to be dated. A particular roc is rich in potassium, so radiometric dating using K-40 is possible. In the surrounding rocks there is 12.1 times more K-40 than its daughter isotope, Ar-40. Determine the age of the rock, and thus, the fossil.

4 820 000 000 years

5. What assumptions must be made in order to accept the answer from 4? Are there any ways to prove that the assumptions are correct?

No decay of daughter, No Ar-40 or K-40 added to rock
Could determine actual decay of daughter and factor this into calculations

6. A wooden bowl found in an ancient Egyptian village contains 40% of the amount of C-14 that there would have been in th air around that time. Determine the age of the bowl.

7600 years

7. What assumptions must be made in order to accept the answer from 4? Are there any ways to prove that the assumptions are correct?

Attachments

Nuclear Decay Graphs.xls